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## **SU(3)–Breaking Effects in Axial–Vector Couplings of Octet Baryons**

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## ABSTRACT

Present evidence on baryon axial–vector couplings is reviewed, the main emphasis being on internal consistency between asymmetry and rate data. A complete account of all *small* terms in the Standard Model description of these latter leads to *both* consistency *and* evidence for breaking of flavour SU(3) in the axial couplings of octet baryons.

## 1. Introduction.

The results we are going to present here constitute a preliminary to a complete study of  $S = -1$  systems, and in particular of the low–energy, coupled  $\bar{K}N$ ,  $\pi\Sigma$ ,  $\pi\Lambda$  channels. We shall concentrate on the description of unpolarized–baryon semi–leptonic decay rates, and in particular on the internal consistency (recently questioned by Jaffe and Manohar<sup>1</sup>) of the present data sets (PDG averages) for both asymmetries and rates.

As a conclusion, we shall present

- a) A new determination of  $|V_{ud}|$  and  $|V_{us}|$ ;
- b) A set of values for the *measured* axial couplings;
- c) Evidence for breaking of flavour SU(3) symmetry in the latter.

Despite recent experimental progress<sup>2</sup>, and the fact that a theoretical analysis of these data is standard business<sup>3</sup>, one still finds in the literature statements which are at best confusing, and sometimes blatantly wrong. For instance, as late as 1992 a review paper<sup>4</sup> is still quoting the analysis by Bourquin *et al.*<sup>5</sup> as the *state-of-the-art* for octet–baryon  $\beta$  decays.

Use of flavour SU(3)  $F$ ,  $D$  constants for the axial charges is still common practice, without or with only handwaving estimates of systematics, often quoting again Bourquin *et al.* as supporting evidence! It is high time to warn that their work<sup>5</sup>, though outstanding for its days, has been (almost) completely superseded by the new evidence<sup>2</sup>, and that use of their  $F$ ,  $D$  values is no longer advisable, since they chose the *wrong* value for  $g_A$ , the nucleon axial coupling.

## 2. The formalism.

In describing the decays  $A \rightarrow B\ell\nu$  for unpolarized initial and final baryons, we use the notations  $\Sigma = M_A + M_B$  and  $\Delta = M_A - M_B$ , express the differential rates in terms of the variable  $q^2$  (where  $q$  is the lepton–pair four–momentum), and the total rates in terms of the adimensional ratios  $\delta = \Delta/\Sigma$  and  $x = m_\ell/\Delta$ .

The vector and axial–vector weak transition currents are fully decomposed<sup>3</sup> via the real–analytic form factors  $F_i(s)$  and  $G_i(s)$  ( $i = 1, 2, 3$ ) as

$$V_\mu(q) = \gamma_\mu \cdot F_1(q^2) + i\sigma_{\mu\nu}q^\nu \cdot F_2(q^2) + q_\mu \cdot F_3(q^2) \quad (1)$$

and

$$A_\mu(q) = \gamma_\mu\gamma_5 \cdot G_1(q^2) + q_\mu\gamma_5 \cdot G_2(q^2) + i\sigma_{\mu\nu}\gamma_5q^\nu \cdot G_3(q^2) , \quad (1')$$

where  $F_3$  and  $G_3$  represent second–class terms and vanish in the flavour–symmetry limit. Since we choose *ab initio* to work in the real world of broken flavour symmetry, their contributions have to be numerically checked (case by case), and we keep them throughout, differing in this from recent, similar works<sup>6</sup>.

Integrating over the leptons’ kinematical variables we obtain the differential rate (to be found, plus a few annoying misprints, at pages 44 and 156 of Pietschmann’s handbook<sup>3</sup>), to which one must add weak and e.m. radiative (and Coulomb) corrections before comparing with the data. The final answer can be cast into the simple expression, after inclusion of all electroweak corrections and integration over the  $q^2$ –variable,

$$\begin{aligned} \Gamma(A \rightarrow B\ell\nu) &= \frac{G_F^2 |\tilde{V}_{KM}|^2}{60\pi^3} \cdot \left(\frac{\Sigma}{2M_A}\right)^3 \cdot \Delta^5 \cdot [F_V^C(\delta, x) + \gamma^2 \cdot F_A^C(\delta, x)] = \\ &= \Gamma_0 \cdot [F_V^C(\delta, x) + \gamma^2 \cdot F_A^C(\delta, x)] , \end{aligned} \quad (2)$$

where  $\gamma = G_1(0)/F_1(0)$  for all cases but  $\Sigma \rightarrow \Lambda e\nu$  transitions, where we define  $\gamma = \sqrt{3/2} \cdot G_1(0)$ , to reduce its SU(3)–limit value to the constant  $D$ . The short–range, electroweak radiative corrections are included defining<sup>7</sup>

$$\tilde{V}_{KM} = V_{KM} \cdot (1 + \delta_W)^{1/2} , \quad (3)$$

with  $\delta_W = 0.0122(4)$  as given by Woolcock<sup>8</sup>; besides, we write<sup>9</sup>

$$F_{V,A}^C(\delta, x) = F_{V,A}(\delta, x) \cdot (1 + \delta_\alpha) + \delta F_{V,A}^C(\delta, x) , \quad (4)$$

with  $\delta_\alpha$  the radiative correction, and  $\delta F_{V,A}^C$  the Coulomb correction to be included for a charged  $B$ , and the uncorrected  $F_{V,A}$  come from integrating the differential rates over  $q^2$ .

In the limit  $\delta \rightarrow 0$ , eq. (3) reduces to  $\Gamma = \Gamma_0 \cdot r_V^C(x) \cdot [1 + 3\gamma^2]$ , good to describe the neutron  $\beta$  decay, but leading to inaccurate results if used for the hyperon  $\beta$  (and muonic) decays. A description of these decays, which aims at both reproducing accurately the data and investigating size and structure of possible SU(3) breaking in their axial couplings, must necessarily account for *all* small terms depending on  $\delta$  in eq. (2), besides the obvious kinematics coming from traces over  $\gamma$ –matrices. Indeed, the latter turn out *not* to dominate these *kinematical* symmetry–breaking effects.

The first of these effects (not always treated consistently in the literature) is the momentum dependence of the form factors. For electric– and magnetic–type, vector and

axial–vector form factors  $F_{1,2}(s)$  and  $G_{1,3}(s)$  (though the last is second–class, the approximation is supported by  $m(a_1) \simeq m(b_1)$  and the strong mixing of the  $Q$ –states in the lowest,  $J^{PC} = 1^{\pm\pm}$  SU(3) meson multiplets), we assume the dipole forms

$$F_1(s)/F_1(0) = F_2(s)/F_2(0) = (1 - \frac{s}{m_V^2})^{-2} \quad (5)$$

and

$$G_1(s)/G_1(0) = G_3(s)/G_3(0) = (1 - \frac{s}{m_A^2})^{-2} , \quad (5')$$

with  $m_V(\Delta S = 1)^2 - m_V(\Delta S = 0)^2 = m_A(\Delta S = 1)^2 - m_A(\Delta S = 0)^2 = m(K^*)^2 - m(\rho)^2$  to ensure the correct variation in the mass scales with the  $\Delta S$  of the weak currents, and  $m_{V,A}(\Delta S = 0)$  are taken from the most recent fits to  $ep \rightarrow ep$ ,  $\bar{\nu}p \rightarrow \ell^+ n$  and  $\nu n \rightarrow \ell^- p$  processes.

With these dipole form for  $G_1$  ( $F_1$ ), we describe the pseudoscalar (scalar) form factors  $G_2$  ( $F_3$ ) with the only inputs of the masses of the pseudoscalar (scalar) states, which we assume to dominate the divergences of the axial–vector (vector) currents. Writing an “extended” PCAC identity like

$$\Sigma \cdot G_1(s) + sG_2(s) = \frac{\sqrt{2}f_P g_{PAB}}{1 - s/m_P^2} + \frac{\Delta_{GT}}{1 - s/m_{P'}^2} \quad (6)$$

(which continues to  $s \neq 0$  the Goldberger–Treiman relation [GTR]), and taking advantage of the *numerical* equality (within errors, quite large in the  $\Delta S = 1$  case) for the GTR discrepancies  $\Delta_{GT}$ ,  $(1 - 2m_P^2/m_A^2) \cdot [1 - \Delta_{GT}/(\Sigma G_1(0))] \simeq 1$ , we rewrite the pseudoscalar form factors  $G_2(s)$  as

$$G_2(s) \simeq \frac{\Sigma G_1(0)}{m_P^2} \cdot [(1 - \frac{2m_P^2}{m_A^2} + \frac{m_{P'}^2 s}{m_A^4})(1 - \frac{s}{m_A^2})^{-2} + \frac{2m_P^2}{m_A^2}(1 - \frac{s}{m_{P'}^2})^{-1}] \cdot (1 - \frac{s}{m_P^2})^{-1} , \quad (7)$$

with the masses  $m_P$ ,  $m_{P'}$  taken from the PDG tables<sup>2</sup> for both  $\Delta S = 0$  ( $\pi$ ,  $\pi'$ ) and 1 ( $K$ ,  $K'$ ) transitions.

In exactly the same way we use the scalar analogue to the GTR

$$\Delta \cdot F_1(s) + sF_3(s) \simeq \frac{\Delta F_1(0)}{1 - s/m_S^2} \quad (8)$$

(but for the  $\Sigma \rightarrow \Lambda e \nu$  case, when  $F_1$  vanishes identically), and we obtain

$$F_3(s) \simeq -\frac{\Delta F_1(0)}{m_V^2} \cdot (2 - \frac{m_V^2}{m_S^2} - \frac{s}{m_V^2}) \cdot (1 - \frac{s}{m_V^2})^{-2} (1 - \frac{s}{m_S^2})^{-1} , \quad (9)$$

still taking the masses  $m_S$  (respectively of the  $a_0$  and  $K_0$  mesons) from the PDG tables<sup>2</sup>. To preserve the scaling with  $\Delta/m_V$  of the (second–class) scalar form factor, we use for the  $\Sigma \rightarrow \Lambda e \nu$  transitions  $F_3(0) \simeq -(\Delta/m_V^2)(2 - m_V^2/m_S^2)\xi$ , with the above  $s$ –dependence, and take as a reasonable *ansatz*  $|\xi| \simeq 1$ .

One can not fix in this way the size of the (second-class) pseudotensor form factor  $G_3(0)$ , for which we assume a dipole behaviour away from  $s = 0$ ; detailed models of the baryons' wavefunctions are needed to give predictions for such a parameter. Indeed, all models based on an SU(6) type of quark wavefunction tend to give small values<sup>10</sup>, and (to the best of our knowledge) no predictions are available from other models. To ensure the symmetry limit, we have chosen to parametrise  $G_3(0)$  as  $G_3(0) = -\Delta G_1(0) \rho/\Sigma^2$ , throwing all our ignorance in the scale parameter  $\rho$ , hopefully such that  $|\rho| \leq O(1)$ . Since in the following we shall use  $\rho = 0$ , one might wonder why include this term at all: we have two motivations, the first that one has always to gauge the *theoretical* systematics (different for  $\rho$  in the rates and in the asymmetries), and the second that consistency requires to consider *all* terms vanishing (as  $\Delta^2$ ) in the symmetry limit.

### 3. Technical remarks and analysis of data.

We turn now to the steps in which we integrate the differential rates and add the Coulomb and radiative corrections: the two steps are *not* independent, since Coulomb corrections are easier to express in terms of a power series in the maximum recoil energy<sup>9</sup> (due to the simplifications occurring in the static limit)  $T_R = \Delta^2 \cdot (1 - x^2)/(2M_A)$ , and thus as a power series in  $\delta^2 \cdot (1 - x^2)$ . It is therefore practical to expand the differential rates in powers of  $\delta$  and then integrate term by term, because the resulting expansion coincides with the analogous one for the Coulomb corrections calculated developing in  $T_R/M_A$  around the static limit.

We write the *uncorrected* functions in eq. (4) as

$$F_V(\delta, x) = F_1(0)^2 \cdot [\phi_1(\delta, x) + 2\kappa^2 \delta^2 \phi_2(\delta, x) + 6\kappa \delta^2 \phi_3(\delta, x) + \\ + \frac{3}{2} (2 - \frac{m_V^2}{m_S^2})^2 \delta_V^4 x^2 \phi_4(\delta, x) + 3 (2 - \frac{m_V^2}{m_S^2}) \delta_V^2 x \phi_5(\delta, x)] , \quad (10)$$

which for  $\Sigma \rightarrow \Lambda e \nu$  transitions reduces to

$$F_V(\delta, x) = 2(\Sigma F_2(0))^2 \delta^2 \phi_2(\delta, x) + \frac{3}{2} (2 - \frac{m_V^2}{m_S^2})^2 \delta_V^4 x^2 \xi^2 \phi_4(\delta, x) , \quad (10')$$

and

$$F_A(\delta, x) = 3 F_1(0)^2 \cdot [\chi_1(\delta, x) + \frac{3}{2} \delta_P^4 x^2 \chi_2(\delta, x) + 3 \delta_P^2 x^2 \chi_3(\delta, x) + \\ + 2\rho^2 \delta^4 \chi_4(\delta, x) + 6\rho \delta^2 \chi_5(\delta, x)] , \quad (11)$$

where (for all cases but  $\Sigma \rightarrow \Lambda e \nu$ , where  $\Sigma F_2(0) = \sqrt{2} \mu_{\Sigma \Lambda}$ )  $\kappa = \Sigma F_2(0)/F_1(0)$  are the SU(3) extensions of the (isovector) magnetic moment used for neutron  $\beta$  decay, and  $\delta_V = \Delta/m_V$ ,  $\delta_P = \Delta/m_P$ .

To avoid an exponential increase of the coefficients in the series with their indices  $n$ , the expansion variable for  $\phi_k$  and  $\chi_k$  must *not* be  $\delta$ , as sometimes stated to justify neglect of some or all the previous, small corrections. One has rather to choose  $\phi_k = \sum_{n=0}^{\infty} f_k^{(n)}(x) \delta_V^{2n}$  (for all  $k$ ) and  $\chi_k = \sum_{n=0}^{\infty} g_k^{(n)}(x) \delta_k^{2n}$ , with  $\delta_k = \delta_A = \Delta/m_A$  for  $k = 1, 4$  and  $5$ , while  $\delta_k = \delta_P$  instead (and thus much larger than the above two) for  $k = 2$  and  $3$ .

To reach high accuracy without too many terms in the series, we use “accelerated convergence”: in simpler terms, we substitute the power series with continued fractions and truncate the latter rather than the former. A third-order approximation turns out more than adequate for the precision required, and we have, for instance,

$$\phi_k(\delta, x) = \frac{f_k^{(0)}(x)f_k^{(1)}(x) - \delta_V^2[f_k^{(0)}(x)f_k^{(2)}(x) - f_k^{(1)}(x)^2]}{f_k^{(1)}(x) - \delta_V^2 f_k^{(2)}(x)}. \quad (12)$$

We lack here the space to tabulate all the coefficients  $f_k^{(n)}(x)$  and  $g_k^{(n)}(x)$ , and we refer the readers to the full version of this work<sup>11</sup>. Neither shall we describe here the techniques for the electroweak corrections terms in eqs. (3,4): again, readers are referred to the details contained in the original papers<sup>7,8,9</sup>.

Using the *corrected* values for  $F_{V,A}^C(\delta, x)$ , and the data listed in the PDG tables<sup>2</sup> for both the rates  $\Gamma(A \rightarrow B\ell\nu)$  and the ratios  $\gamma = G_1(0)/F_1(0)$  obtained from the asymmetries, one can follow two paths in the use of eq. (3): a) extract the CKM matrix elements  $|V_{ud}|$ ,  $|V_{us}|$  from the *experimental* values of  $\Gamma$  and  $\gamma$ , or b) use CKM unitarity, the bounds on  $|V_{ub}|$  from charmless  $B$ -meson decays and the value for  $|V_{ud}|$  from superallowed nuclear  $\beta$  decays (assuming only three families of “light” quarks), to extract the absolute values  $|\gamma|$  from the rates  $\Gamma$ . In both cases we have to invoke both the Ademollo–Gatto theorem<sup>12</sup> for the vector *charges*  $F_1(0)$  (note that arguments about its violation do not always separate the *charges* from the *total vector couplings*, a dangerous attitude when working with models for baryon states *at rest*), and the approximation  $\rho \simeq 0$  (to make connections with the asymmetry values for  $\gamma$ , all derived under this assumption).

**Table I**  
**Moduli of the KM-matrix elements**

Decay	from data	recommended by PDG
$\Delta S = 0$ :		
$n \rightarrow pe\bar{\nu}_e$	$0.97310 \pm 0.00213$	$0.9747 - 0.9759$
$\Delta S = 1$ :		
$\Lambda \rightarrow pe\bar{\nu}_e$	$0.22478 \pm 0.00348$	
$\Lambda \rightarrow p\mu^-\bar{\nu}_\mu$	$0.2305 \pm 0.0259$	
$\Sigma^- \rightarrow ne\bar{\nu}_e$	$0.22308 \pm 0.00480$	
$\Sigma^- \rightarrow n\mu^-\bar{\nu}_\mu$	$0.21009 \pm 0.00972$	
$\Xi^- \rightarrow \Lambda e\bar{\nu}_e$	$0.23092 \pm 0.00948$	
average	$0.22376 \pm 0.00259$	$0.218 - 0.223$

The results on CKM matrix elements are listed in table I: the value  $|V_{ud}| = 0.9731(21)$  agrees with that from superallowed Fermi transitions in nuclei<sup>13</sup>,  $|V_{ud}| = 0.9740(5)$ , on the *low side* of the PDG “adjusted” range. This is because the PDG gives more weight to the *theoretical* analysis of  $K_{\ell 3}$  decays by Leutwyler and Ross<sup>14</sup> than to the data from baryon  $\beta$  decays<sup>15</sup>; our evaluation, based on better data (and a more complete analysis) than the original one, but consistent with their (revised) findings<sup>15</sup>, gives  $|V_{us}| = 0.2238(26)$  (and

$|V_{ud}|^2 + |V_{us}|^2 = 0.9970(53)$ ), this time on the *high side* of the PDG range, in accord with expectations from CKM unitarity for three families, found valid to better than  $1\sigma$ , and the estimate  $|V_{ub}| = (4 \pm 2) \times 10^{-3}$ .

We next choose to sit on the low side of the PDG range for  $|V_{ud}|$  (i.e. right on the value from Fermi transitions) and thus obtain the values for  $|\gamma|$  listed in table II. Defining a consistency parameter  $\chi_{cons}^2$  as

$$\chi_{cons}^2 = \sum \frac{2(\gamma_{rate} - \gamma_{asym})^2}{\sigma_{rate}^2 + \sigma_{asym}^2}, \quad (13)$$

where we assume for both  $\gamma$ 's the same sign, we find a definite decrease of this parameter with  $|V_{ud}|$ , of more than five units over the PDG range, to be compared with a minimum of 3.18 at our chosen value, and we claim that *all* baryon  $\beta$  decays require to reduce  $|V_{ud}|$  and raise  $|V_{us}|$ , and advise *against* using the PDG “central values” when precise estimates (at percent level or better) are required. Note also that baryonic  $\beta$  decay data (including superallowed Fermi nuclear decays) have now an *overall* quality much better than  $K_{\ell 3}$  ones: in our opinion the error quoted by Leutwyler and Ross<sup>14</sup> for their estimate of  $|V_{us}|$  was underestimated by at least a factor 2 (as already advocated by Paschos and Turke<sup>16</sup>).

**Table II**  
**Axial couplings**

Decay	from rates (moduli)	from asymmetries (PDG)	SU(3) fits
$\Delta S = 0$ :			
$n \rightarrow pe\bar{\nu}_e$	$1.2548 \pm 0.0018$	$1.2573 \pm 0.0028$	1.2552
$\Sigma^- \rightarrow \Lambda e\bar{\nu}_e$	$0.7223 \pm 0.0173$	--	0.7865
$\Sigma^+ \rightarrow \Lambda \bar{e}\nu_e$	$0.750^{+0.089}_{-0.101}$	--	0.7865
$\Sigma^- \rightarrow \Sigma^0 e\bar{\nu}_e$	--	--	0.4687
$\Xi^- \rightarrow \Xi^0 e\bar{\nu}_e$	$< 2 \times 10^3$	--	0.7308
$\Delta S = 1$ :			
$\Lambda \rightarrow pe\bar{\nu}_e$	$0.7251 \pm 0.0112$	$0.718 \pm 0.015$	0.7308
$\Lambda \rightarrow p\mu^-\bar{\nu}_\mu$	$0.756^{+0.128}_{-0.154}$	--	0.7308
$\Sigma^- \rightarrow ne\bar{\nu}_e$	$0.3377^{+0.0217}_{-0.0232}$	$-0.340 \pm 0.017$	-0.3178
$\Sigma^- \rightarrow n\mu^-\bar{\nu}_\mu$	$0.2466^{+0.0664}_{-0.0928}$	--	-0.3178
$\Xi^- \rightarrow \Lambda e\bar{\nu}_e$	$0.2600^{+0.0411}_{-0.0490}$	$0.25 \pm 0.05$	0.2065
$\Xi^- \rightarrow \Lambda\mu^-\bar{\nu}_\mu$	$0.763^{+0.470}_{-0.763}$	--	0.2065
$\Xi^- \rightarrow \Sigma^0 e\bar{\nu}_e$	$1.263^{+0.143}_{-0.161}$	--	1.2552
$\Xi^- \rightarrow \Sigma^0 \mu^-\bar{\nu}_\mu$	$< 37$	--	1.2552
$\Xi^0 \rightarrow \Sigma^+ e\bar{\nu}_e$	$< 2.76$	--	1.2552
$\Xi^0 \rightarrow \Sigma^+ \mu^-\bar{\nu}_\mu$	$< 30$	--	1.2552

The SU(3)–symmetry fit in table II leads to a  $\chi^2$  of 21.98 versus 12 d.o.f., not unacceptable from a purely statistical point of view; however, by comparing axial charges

from the fit with those averaged from the data *à la* PDG, one can see that almost all the  $\chi^2$  comes from the  $\Sigma \rightarrow \Lambda$  transition, with lesser amounts contributed by the  $\Delta S = 1$  transitions other than  $\Sigma^- \rightarrow n$ , while of course the accurate  $n \rightarrow p$  one acts as a constraint on the sum  $F + D$ .

We thus conclude that, unless the measurements on  $\Sigma \rightarrow \Lambda e \nu$  transitions are redone, yielding rates deviating upward of previous measurements by several standard deviations, there is solid evidence for first-order SU(3) violation in the axial charges, which, we repeat again, *do not* show any sign of internal inconsistency warranting an increase in their *experimental* errors (if one uses state-of-the-art theoretical analysis, not badly approximated and completely outdated formulæ, as for instance was done by Jaffe and Manohar<sup>1</sup>).

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